Learning Together Alliance

LEARNING TOGETHER

Maths Alliance
Mathematics Mastery
Calculation Policy
September 2019

SANDOWN PRIMARY SCHOOL AND NURSERY

THE BEST WE CAN BE

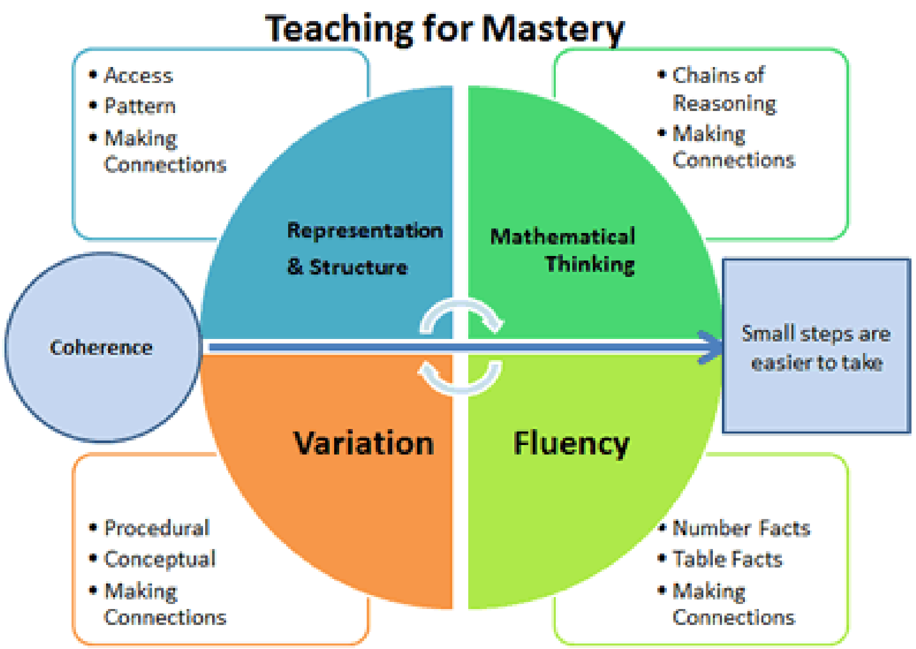
Person responsible for the policy	Katrina Pounder
Date reviewed and shared with staff	Emailed
Date to next be reviewed by staff	September 2019
Date ratified by the Governing Body	n/a
Date to be reviewed by the Governing Body	n/a

Mathematics Mastery

The Mathematics Mastery approach is the belief that **all pupils have the potential to succeed**. They should have access to the same curriculum content, rather than being extended with new learning; they should therefore deepen **their conceptual understanding by tackling challenging and varied problems**. Calculation strategies are not learnt by rote but through understanding of procedures using concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught, including the Five Big Ideas for Teaching Mastery, and used in line with the requirements of the 2014 Primary National Curriculum.

FIVE BIG IDEAS FOR TEACHING FOR MASTERY

A central component in the NCETM programmes to develop Maths Mastery has been around the Five Big Ideas, drawn from research evidence, underpinning teaching for mastery. This is the diagram used to help bind these ideas together. [NCETM]



Coherence
Connecting new ideas to concepts that have already been understood, and ensuring that, once understood and mastered, new ideas are used again in next steps of learning, all steps being small steps

Representation and Structure
Representations used in lessons expose the mathematical structure being taught, the aim being that students can do the maths without recourse to the representation

Mathematical Thinking
If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the student: thought about, reasoned with and discussed with others

Fluency
Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics

Variation
Varying the way a concept is initially presented to students, by giving examples that display a concept as well as those that don't display it. Also, carefully varying practice questions so that mechanical repetition is avoided, and thinking is encouraged. (NCETM)

Mathematical Language

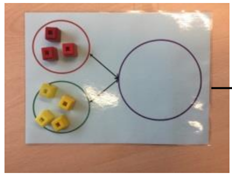
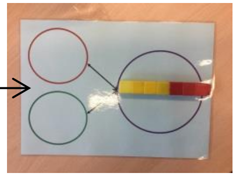
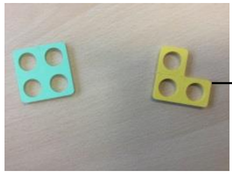
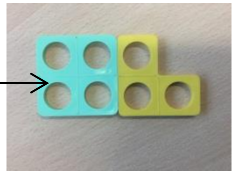
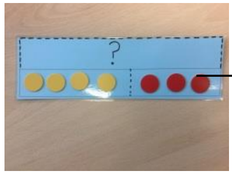
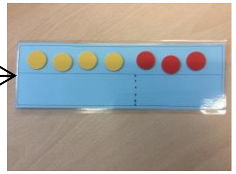
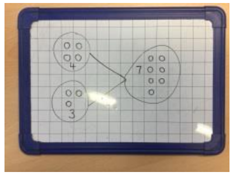
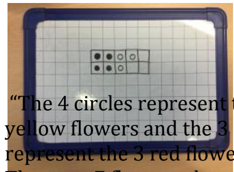
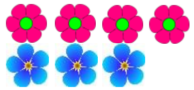
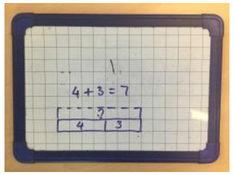
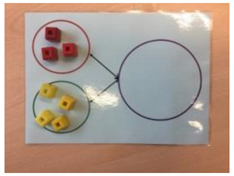
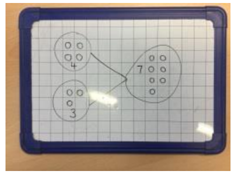
The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (*reasoning*). It is essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. The school agreed list of terminology is located at Appendix A to this document.

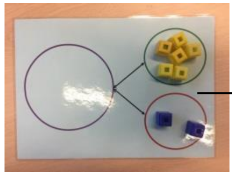
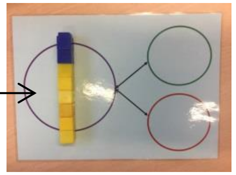
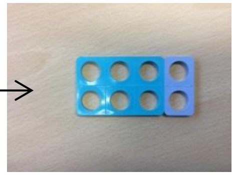
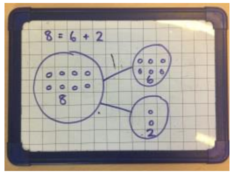
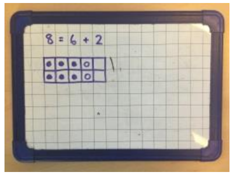
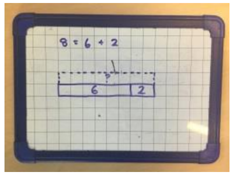
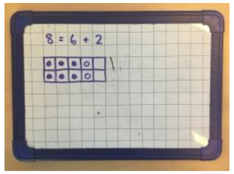
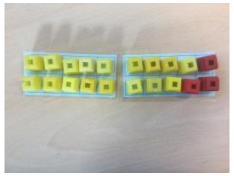
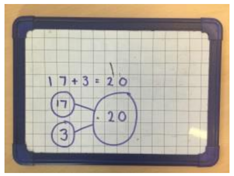
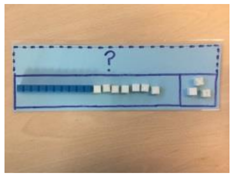
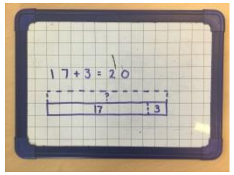
How to use the policy

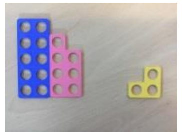
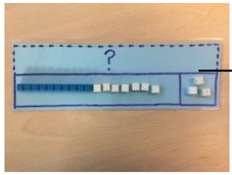
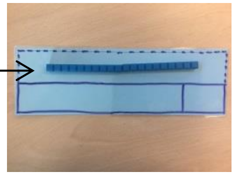
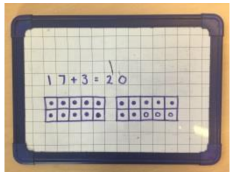
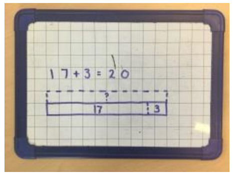
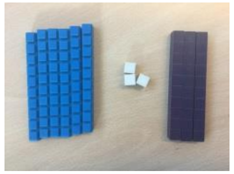
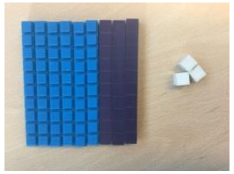
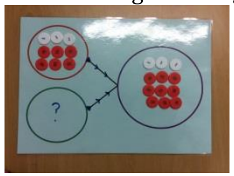
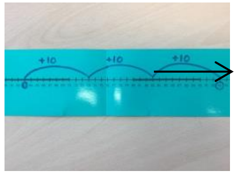
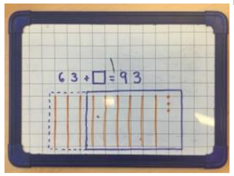
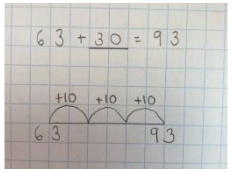
This mathematics policy is a guide for all staff in the Learning Together Alliance and has been adapted from work by the NCETM. It is set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when to consolidate existing skills or whether to move onto the next concept. However, the **focus must always remain on breadth and depth rather than accelerating through concepts.** Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems.

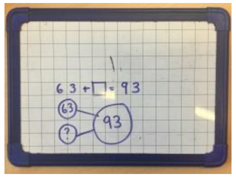
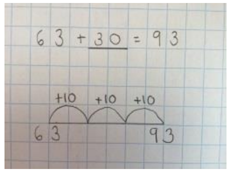
For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

Addition

Objective and Strategies	**Concrete** Children should be taught to use a range of resources to represent one calculation. Children should verbalise their use of resources using the appropriate mathematical language.	**Pictorial** Children should be taught to draw pictorial representations independently. Children can use different colours or symbols to distinguish between the different parts in the number sentence.	**Abstract** The abstract should be recorded alongside the concrete and pictorial.
Adding with one-one correspondence 4+3=7 There are 3 red flowers and 4 yellow flowers. How many flowers are there altogether? *Joining two groups of objects together to form a whole. Children then count using one-one correspondence by touching each object and saying the number name.*	      	 "4 parts plus 3 parts equals 7. The whole equals 7."  "The 4 circles represent the 4 yellow flowers and the 3 circles represent the 3 red flowers. There are7 flowers altogether" "4 flowers plus 3 flowers equals 7 flowers."  "4 is a part and 3 is a part. The whole is 7." 	  Children can record multiple number sentences from one representation e.g. 4+3=7 3+4=7 7=4+3 7=3+4 Alongside teaching addition children should be taught the inverse operation within the same lesson. If there are 7 flowers altogether and I took away 4 yellow flowers, how many red flowers would there be left? Children would then record the subtraction number sentences appropriate to the representation e.g. 7-4=3 3=7-4

Counting on to add 1, 2 or 3 (or a multiple of 10, 100 or 1,000) 8=6+2 What is the total of 6 footballs and 2 footballs? *As a strategy, this should be limited to adding small quantities only with pupils understanding that counting on from the greater number is the most efficient.* *Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting on as their main strategy.*	      	 "The whole equals 8. 6 parts add 2 parts equals 8."  "The 6 coloured circles represent 6 footballs. The 2 empty circles represent 2 more footballs. There are 8 footballs altogether." "6 footballs plus 2 footballs equals 8 footballs."  "8 is the whole. 6 parts add 2 parts equals 8." 	  Children can record multiple number sentences from one representation e.g. 8=6+2 8=2+6 6+2=8 2+6=8 Alongside teaching addition children should be taught the inverse operation within the same lesson. If there are 8 footballs altogether and I took away 6 footballs, how many footballs would there be left? Children would then record the subtraction number sentences appropriate to the representation e.g. 8-6=2 2=8-6
Number Bonds to 10, 20 and 100. 17+3=20	  When children can see that a tens frame is full,	 "17 is a part and 3 is a part. Altogether the whole equals 20."	  Children can record multiple number sentences

If I have 17p and I am given 3p. How much money do I have now? *Children should have a secure understanding of 10 and what ten looks like in different representations. Children need to be taught the different combinations of numbers that make 10, 20 and 100. Children need to become fluent with these facts.*	children should not count in ones. They should count in tens (and ones if neccessary).    When children are working with Numicon and can see an equivalence to 10 (e.g. the 7+3), children should use a tens plate to represent the equivalence.   When children are working with base 10 and they need to rename, children should confidently rename 10 ones into 1 ten.	 "The 17 coloured circles represent 17p. The 3 empty circles represent the 3p. The total equals 20p."  "The whole is unknown so we had to add the 2 parts together. We need to add 17 and 3 to make 20."  "17p plus 3p equals 20p."	from one representation e.g. 17+3=20 3+17=20 20=17+3 20=3+17 Alongside teaching addition children should be taught the inverse operation within the same lesson. If I had 20p and I spent 3p, how much money would I have left? Children would then record the subtraction number sentences appropriate to the representation e.g. 20p-3p=17p 17p=20p-3p
Adding powers of 10 (10, 100, 1,000, 10,000 etc.) or a multiple of 10. $63+\boxed{}=93$ I have 63 pencils but I need 93. How many more do I need? I have 93 pencils in total. 63 are blue,	  Dienes apparatus allows children to see that when adding in powers of ten, certain columns remain unchanged. This builds up a conceptual understanding of adding in powers of 10.   When children are working with the part-part-whole diagram or the bar	 "I need to get from 63 to 93 to find the unknown. I can jump in tens. I have added 3 tens to 63 to get to 93. 63+30=93."   "I know that the whole is 93 so I can draw this in base 10. If one of the parts is 63 then the remaining part is 30."	  Children can record multiple number sentences from one representation e.g. 63+30=93 30+63=93 93=63+30 93=30+63 Alongside teaching addition children should be taught the inverse operation within the same

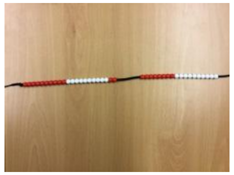
how many are green? *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a ten and not a one that is being added.* *It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.*	model, it allows children to be able to see the unknown and use counting on skills to work out the missing addend.	 "63 is a part and the other part is unknown. The whole is 93. I need to add 30 to 63 to find the whole." This pictorial representation does not solve the calculation. Children will still need to use resources or draw a picture (see other examples). Children may calculate the answer mentally by counting on from 63 to 93 in tens.   The place value chart shows the tens counter moving 3 tens places to represent the missing addend being 30.  The number line shows 3 jumps of 10 to show that the missing addend is 30. Children should recognise that the ones digit is the same so the tens column is changing.	lesson. If I had 93 pencils and 63 were blue, how many pencils would be green? Children would then record the subtraction number sentences appropriate to the representation e.g. 93-63=30 30=93-63

Combining to make a multiple of 10

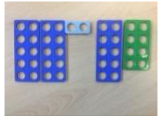
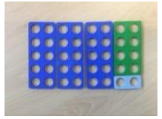
$$22+18=40$$

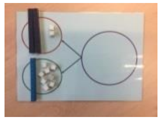
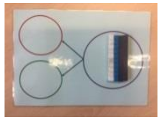
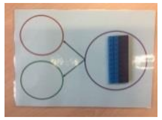
There are 22 red cars and 18 blue cars. How many cars are there altogether?

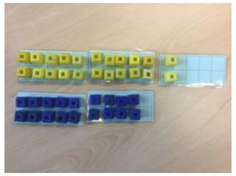
Children will be using their understanding of adding multiples of ten as well as their number bonds to combine numbers to make a multiple of ten e.g. In this example, children should add 2 tens and 1 ten to make 3 tens and 2 ones and 8 ones to make 10 ones. Children then rename their 10 ones to one ten to create 40 in total.




The colours of the beads on the bead string make it clear how many more need to be added to make ten.

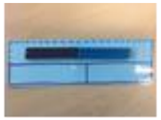



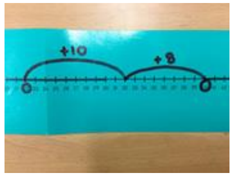





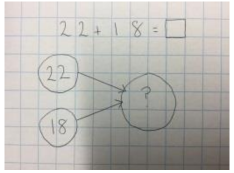


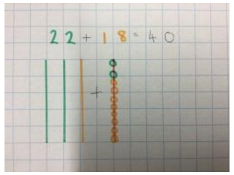

The empty spaces on the tens frame make it clear how many more are needed to make ten.



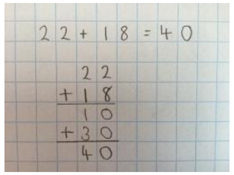


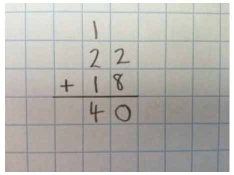






"22 is a part. 18 is a part. 2 and 8 make 10. 20 and 10 make 30. 30 and 10 make 40. The whole is 40."
These pictorial representations show different ways that 22 and 18 could be combines. The bar model and the part-part-whole model do not show the sum of the numbers but they do show children to combine to make a total amount.
The drawing of dienes allows the children to see how ten is made. This picture should be drawn alongside the concrete resources.





These two written methods show how 22 + 18 could be solved. Both the expanded column method and the compact column method need to be taught. These written methods are progressive; once children have a secure understanding of place value, they can move onto the compact column method. Children should not 'get stuck' in the expanded method.

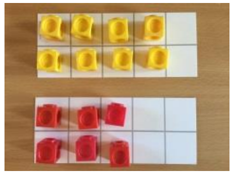
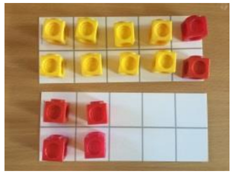
Bridging through 10

Adding 2 single digits

14=8+6

How many sides are there on an octagon and a hexagon?

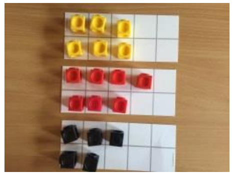
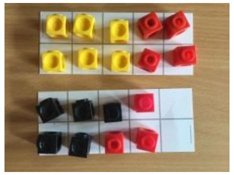
In KS1 this is often referred to as the 'magic ten'. 'Magic ten' encourages the children to make ten when bridging.

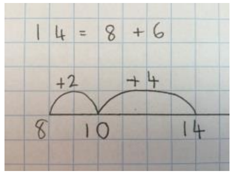
The tens frame allows children to see how many more they need to make 10. This allows them to bridge 10 with a clearer understanding. Children can see that they have 8 cubes and need to redistribute 2 from the 6 to make 10. Children can then add 10+4 to find the sum.



The bead string allows children to see that they need 2 more beads to make 10. They can then see that they have 10+4 to find the sum.

From making the three addends on three separate tens frames, children should be encouraged to look at how they can make 10 within the calculation. In the second picture the 7 has been redistributed into 4 and 3. The 6 and 4 make 10 and the 5 and 3 make



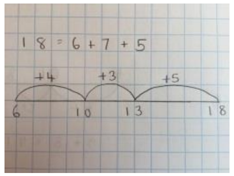
Children could:

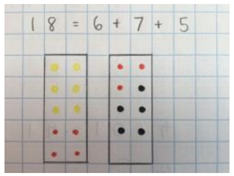
- Draw a number line
- Draw a part-part-whole diagram
- Draw dienes
- Or, draw a bar model.

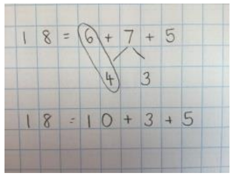
From their pictures the children should be able to verbalise:

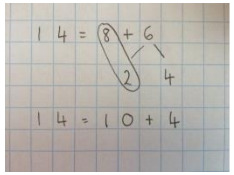
"8 is a part and 6 is a part. I need to partition 6 into 2 and 4 because 8+2=10 and 4 more makes 14."

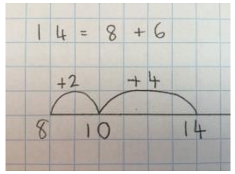
"8 is a part and 6 is a part. Altogether the whole equals 14."









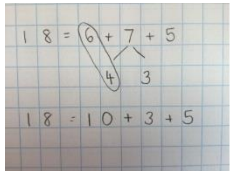


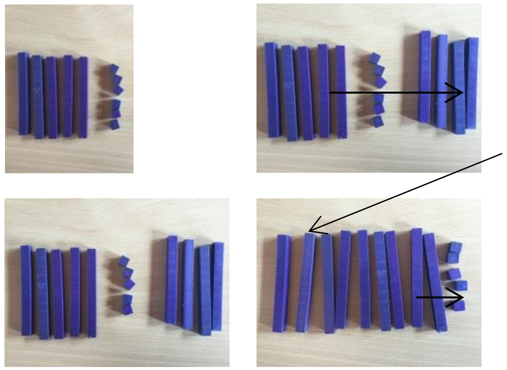
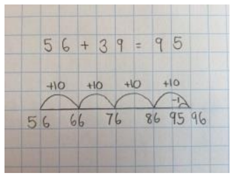
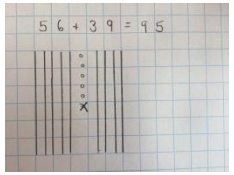
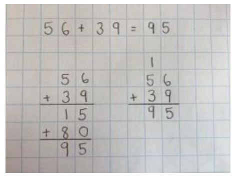
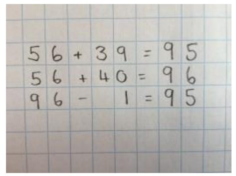
Children can record multiple number sentences from one representation e.g.

14 = 8 + 6
14= 6+ 8
6 + 8 = 14
8 + 6 = 14

Alongside teaching addition children should be taught the inverse operation within the same lesson. 14 take away 6 equals 8. The difference between 14 and 8 is 6.

14-6=8



Adding 3 single digits 18=6+7+5 *There are 6 red pencils, 7 blue pencils and 5 green pencils. How many pencils are there altogether?* *Pupils should be encouraged to 'look for ten' within their calculation.*	8. The sum is 18.	All pictorial representation show the same as the concrete representation. The seven has been partitioned into 4 and 3 so that 10 can be made. "I have partitioned 7 into 4 and 3. 6 and 4 make ten, 3 and 5 make 8. 10 and 8 make 18. The whole is 18."	Children can record multiple number sentences from one representation e.g. 18 = 6 + 7 + 5 6 + 7 + 5 = 18
Compensating to add 56+39=___ I have £56 in my money box. Then I received £39 for my birthday. How much money do I have? *Children need to have a secure*	 The first image shows 56. The second image shows 56+40. The third image shows 1 has been subtracted from 59 to	  All pictorial representations show the same as the concrete representation. Forty has been added to 56 and then one has been subtracted. "I have added 40 onto 56 because 40 is	  Children can record multiple number sentences from one representation e.g. 56 + 39 = 95 95 = 56 +39 Alongside teaching addition children

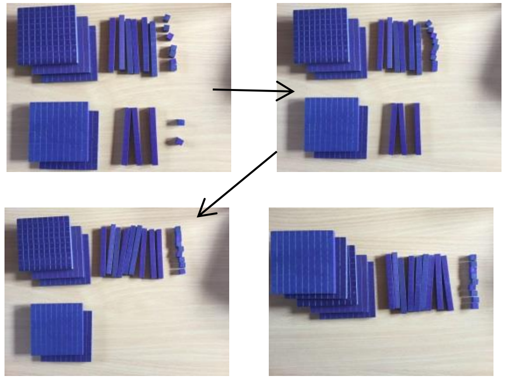
understanding that 9 is 1 less than 10 to be able to use this method. *Children will add ten before compensating by subtracting 1.* *Children should then apply this strategy to add 19, 29, 39 (etc.) or to add 8, 18, 28, 38 (etc.)* Children should still using compensation strategies with greater numbers e.g. **384 +90 =** **384 + 99=** **10,587 + 9,990 =** 10,587 fans supported the red team. 9,990 fans supported the blue team. How many supporters were there? Children should use a near multiple of 10, 100, 1,000 or 10,000 and then readjust by compensating.	make 58 as this compensates for the additional one that has been added to 39. The final image shows the sum is 95.	1 more than 39. I have then subtracted 1 from 96 to compensate for the extra 1 that I added. The total is 95."	should be taught the inverse operation within the same lesson. When compensating children would subtract 40 and add one back on to compensate for the additional 1 they have subtracted. 95-39=56

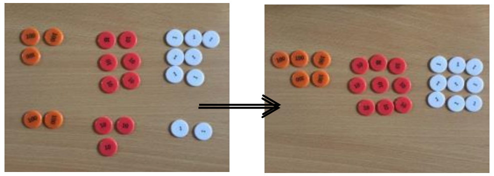
Adding using partitioning with no renaming

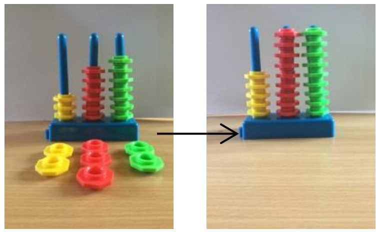
367+232=

Two schools met for a sports competition. One school had 367 children and the other school had 232 children. How many children attended the event?

In theory this is a mental strategy, however children need to be taught how to add without renaming by using the C-P-A approach. Once children are confident with this strategy, it should become a mental calculation.

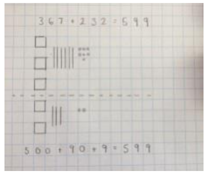


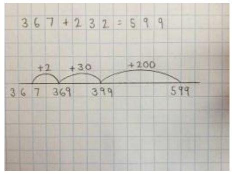


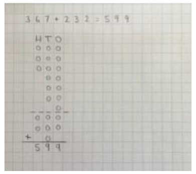


These concrete resources show that the ones column has been added first, then the tens column and finally the hundreds column.

The place value chart shows that the ones counter, the tens coutner and the hundreds counter have moved to the right which symbolises addition.

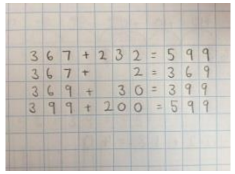


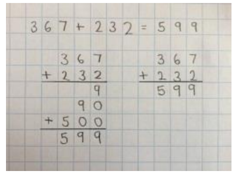




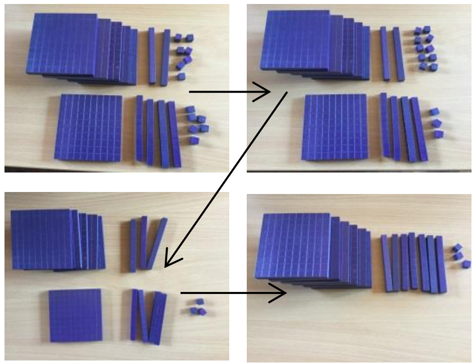
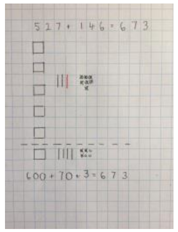
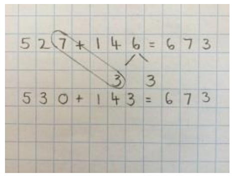
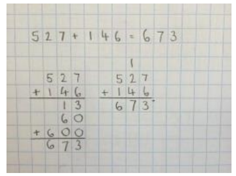
All pictorial representations show the same as the concrete representation. 232 has been added to 367 by counting on in ones, tens and then hundreds. Children should be taught to add in ones, tens and then hundreds to support them when moving into the abstract representation of a written method.

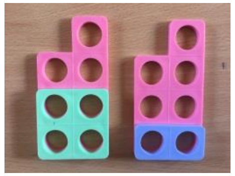
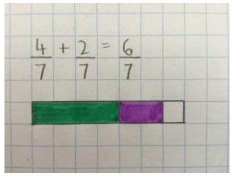
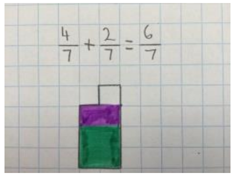
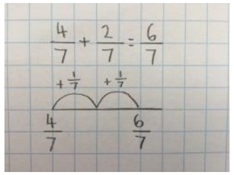
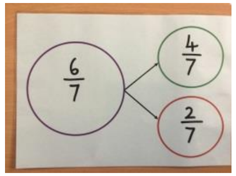
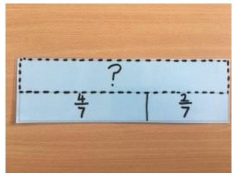
“I have added 232 to 367 by partitioning 232 into 200, 30 and 2. I have started with the ones column in case I need to rename in any columns. In this question, I do not need to rename because none of the columns total more than 9.”





These two written methods show how 367 + 232 could be solved. Both the expanded column method and the compact column method need to be taught. These written methods are progressive; once children have a secure understanding of place value, they can move onto the compact column method.

	Children would move the ones counter first, then the tens and finally the hundreds.		
Addition with renaming 527+146= Yellows gained 527 team points and Blues gained 146 team points. How many team points were gained altogether? *It is important for children to use the correct mathematical language when calculating using this strategy e.g. 7 ones add 6 ones equals 13 ones so I need to rename 10 ones to 1 ten and put it in the tens column.*	 	  All pictorial representations show the same as the concrete representation. 3 has been added to 527 to make 530. The remaining 143 has been added to 530 to make 673.	 These two written methods show how 527 + 146 could be solved. Both the expanded column method and the compact column method need to be taught. These written methods are progressive; once children have a secure understanding of place value, they can move onto the compact column method.

	    These concrete resources show that 3 ones from 146 have been added to the 527 to make 530('magic ten'). This leaves 143 to be added. The ten ones have been renamed as one ten. Finally, the 143 is added to the 530.		
Adding fractions with the same denominator within one whole $$\frac{4}{7}+\frac{2}{7}=\frac{6}{7}$$ Sam ate $\frac{4}{7}$ of a chocolate bar and Vivian ate $\frac{2}{7}$. How much did they eat altogether? EXT: How much of the chocolate bar is	  For children to be able to add fractions within one whole, they need to show each addend and then combine them to find the total. 	   These pictorial representations replicate the concrete representation of adding fractions with the same denominator. The number line shows that children are adding two sevenths by adding one seventh at a time. "I know that 4+2=6 so $\frac{4}{7}+\frac{2}{7}=\frac{6}{7}$"	  These representations are abstract because nothing about the numbers within the bar model or part-part-whole model represent the value of the addends. Children will record the number sentence alongside their pictorial representation in their books. $\frac{4}{7}+\frac{2}{7}=\frac{6}{7}$ or $\frac{6}{7}=\frac{2}{7}+\frac{4}{7}$

<table>
<tr>
<td>left?

Pupils should be taught to practise adding fractions with the same denominator to become fluent through a variety of increasingly complex problem within one whole. Children should be using fluent recall of addition facts to calculate the total.</td>
<td>The Numicon and Cuisenaire show that the denominator is sevenths and the numerators are 4 and 2. Both concrete representations reveal that $\frac{6}{7}$ is one seventh less than the whole.

Children should be applying their number bonds to 7 when adding fractions with the same denominator.</td>
<td></td>
<td></td>
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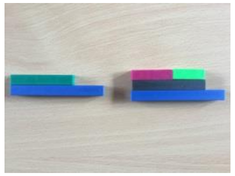
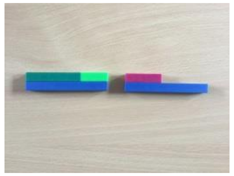
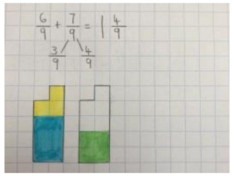
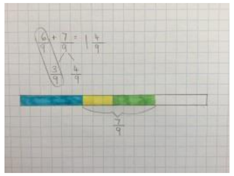
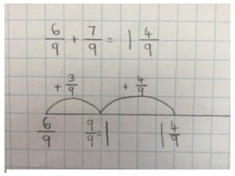
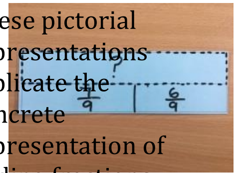
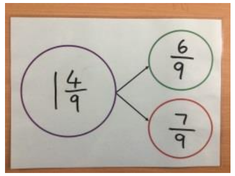
<tr>
<td>Adding fractions with the same denominator greater than one whole.

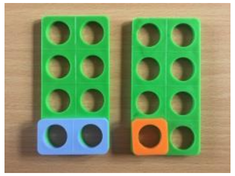
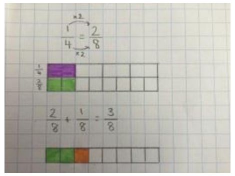
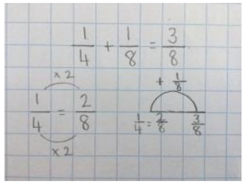
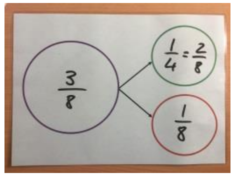
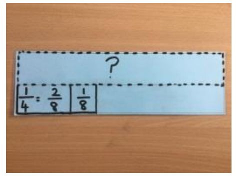
$$\frac{6}{9} + \frac{7}{9} = 1\frac{4}{9}$$

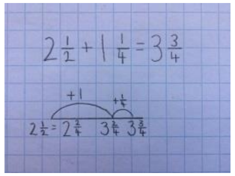
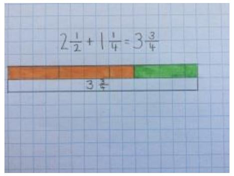
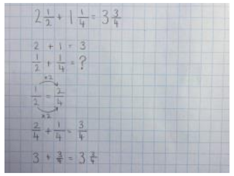
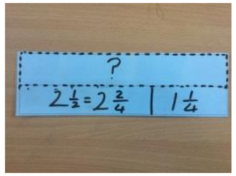
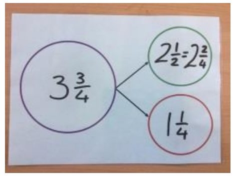
Elsie eats $\frac{6}{9}$ of her pizza. Jemma eats $\frac{7}{9}$ of her pizza. How much pizza did they eat altogether?
EXT: Could they have shared a pizza?</td>
<td> 

 
To be able to use concrete apparatus when adding fractions with the same denominator greater than one whole, children need to use their</td>
<td>


"I know that I need to partition 7 ninths so that I can make one whole with 6 ninths and 3 ninths. I would then have 4 more ninths to add. The total is $1\frac{4}{9}$.</td>
<td>These pictorial representations replicate the concrete representation of adding fractions with the same denominator greater than one whole.
The number line shows that children are adding 3 ninths than then 4 ninths.
They have added 7 ninths altogether.
 
These representations are abstract because nothing about the numbers within the bar model or part-part-whole model represent the value of the addends.

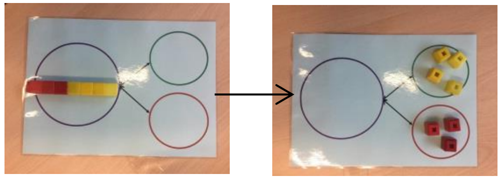
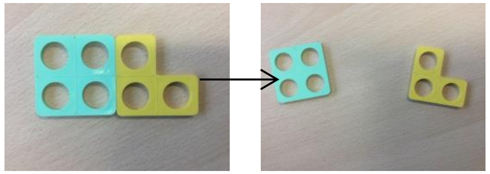
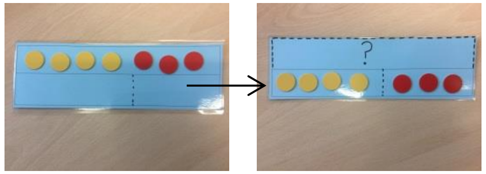
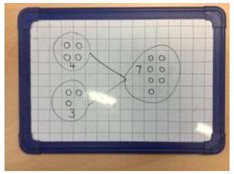
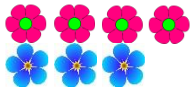
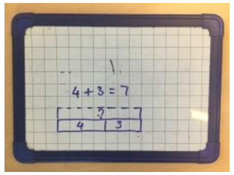
Children will record the number sentence alongside their pictorial representation in their books.

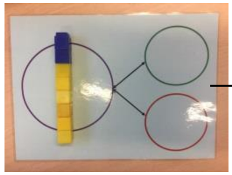
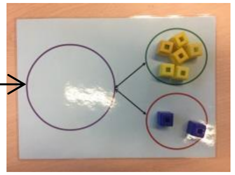
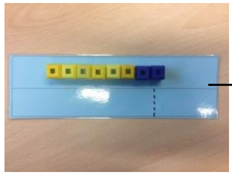
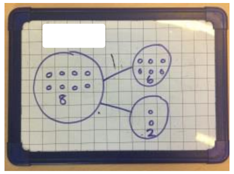
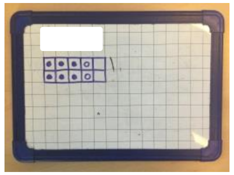
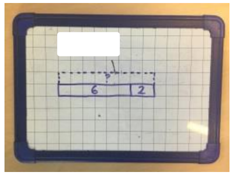
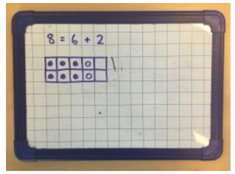
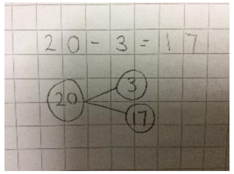
$\frac{6}{9} + \frac{7}{9} = 1\frac{4}{9}$ or $1\frac{4}{9} = \frac{6}{9} + \frac{7}{9}$</td>
</tr>
</table>

Pupils should be taught to practise adding fractions with the same denominator to become fluent through a variety of increasingly complex problem greater than one whole. Children should be using fluent recall of addition facts to calculate the total.	understanding of bridging to bridge through the whole, in this case ninths. In this example, the $\frac{7}{9}$ has been partitioned into $\frac{3}{9}$ and $\frac{4}{9}$ to bridge the whole. This then shows that the sum is $1\frac{4}{9}$.		
Adding fractions with the same denominator and multiples of the same number e.g. $$\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$$ $\frac{1}{4}$ of the class have blonde hair. $\frac{1}{8}$ of the class have black hair. How many children have blonde or black hair? *This strategy focuses on pupils*	   When children move onto adding fractions with different denominators, the concrete resources become more abstract because children need to convert the fractions into common denominators before they can calculate e.g. in this case eighths is the common denominator $\frac{2}{8}=\frac{1}{4}$.	  "I have converted $\frac{1}{4}$ into $\frac{2}{8}$ so that I can show the equivalence of $\frac{2}{8}$ and $\frac{1}{4}$. I can then add $\frac{2}{8}+\frac{1}{8}$ to find the sum".	  These representations are abstract because nothing about the numbers within the bar model or part-part-whole model represent the value of the addends. Children will record the number sentence alongside their pictorial representation in their books.

understanding of equivalence. Children will need to convert fractions to be able to add them fluently. This then follows the strategy where children are adding fractions of the same denominator (see above).			
Adding fractions with different denominators or mixed number. $$2\frac{1}{2} + 1\frac{1}{4} = 3\frac{3}{4}$$ *It took $2\frac{1}{2}$ hours to bake a cake and $1\frac{1}{4}$ hours to ice the cake. How long did it take me to make the cake?* *Children need to have a secure understanding of equivalence as they will need to identify equivalent fractions with the same denominator.*	When calcuations move into higher-order thinking, the concrete becomes the abstract. It is easier for children to represent this calcuation by using pictorial or abstract representations. Children could use place value counters to represent the fractions $2\frac{1}{2} + 1\frac{1}{4}$. This relies on them having a secure understanding of place value and decimal/fraction equivalence.	  "To add $1\frac{1}{4}$ to $2\frac{1}{2}$, I need to add $\frac{1}{4}$ to $\frac{1}{2}$, which I know equals $\frac{3}{4}$. I can then add the ones digits together to find the sum $3\frac{3}{4}$. If children draw a bar model, the bars need to be roughly proprotionate to show the worth of each addend.	   The abstract representations show that there are two parts which are combined to make a whole. Conversion arrows are used to find a common denominator before being able to add. Once children are fluently able to recall fraction equivalents, they can move away from conversion arrows.

Subtraction

Objective and Strategies	**Concrete** Children should be taught to use a range of resources to represent one calculation. Children should verbalise their use of resources using the appropriate mathematical language.	**Pictorial** Children should be taught to draw pictorial representations independently. Children can use different colours or symbols to distinguish between the different parts in the number sentence.	**Abstract** The abstract should be recorded alongside the concrete and pictorial.
Subtracting with one-one correspondence **7 – 3 = 4** There are 7 flowers altogether. 3 flowers wilt and need to be taken out of the vase. How many flowers are left? *Children should subtract one group of 3 from the whole (7). Children should then use one-to-one correspondence and say the number names to find how many are left?*	   	 "The whole is 7. 3 is a part and 4 is a part. 7 – 3 = 4."  "There are7 flowers altogether. The 4 circles represent the 4 yellow flowers and the 3 circles represent the 3 red flowers."  "7 flowers subtract 3 flowers equals 4 flowers."  "The whole is 7. 3 is a part and 4 is a part."	Children can record multiple number sentences from one representation e.g. 7 – 3 = 4 7 – 4 = 3 3 = 7 – 4 4 = 7 - 3 Alongside teaching subtraction children should be taught the inverse operation within the same lesson. There are 3 red flowers and 4 yellow flowers. How many flowers are there altogether? Children would then record the addition number sentences appropriate to the representation e.g. 4 + 3 = 7 3 + 4 = 7 7 = 4 + 3 7 = 3 + 4

Counting back to subtract 1, 2 or 3 (or multiple of 10, 100 or 1,000) 8 – 2 = 6 8 sweets were in a bag. 2 sweets were taken out. How many sweets were left in the bag? *Pupils should be encouraged to use number bonds as their main strategy e.g. knowing that 8-2=6 so when they calculate within 20 they can apply their facts to calculations such as 18-2=16.*	Create the minuend using mathematical resources and then subtract the subtrahend to find the difference.       	 “The whole equals 8.2 is a part and the other part is 6.” “  The 8 circles represent the sweets in the bag. The two empty circles represent the sweets that have been taken out. There are 6 sweets left in the bag.”  “8 is the whole subtract 2 parts equals 6 parts left.”	  Children can record multiple number sentences from one representation e.g. 8-2=6 8-6=2 6=8-2 2=8-6 Alongside teaching subtraction children should be taught the inverse operation within the same lesson. If the problem was, “what is the total of 2 sweets and 6 sweets?” children would then record the addition number sentences appropriate to the representation e.g. 8=6+2 8=2+6 6+2=8 2+6=8
Numbers bonds to subtract 20 – 3 = 17 3 fish swim away from a shoal of 20. How many fish are left? *Children should know that 3+7=10 so 20-3=17 so should apply this when subtracting.*	Create the minuend with resources and subtract the subtrahend to show the difference. Children should be able to apply their knowledge of 10-3=7 to 20-3=17 These resources could be used to represent the calculation:	 Children can use the part and whole diagram to show that 3+17=20 so 20-3=17 “20 is the whole so it is the minuend. 3 is a part of the whole so it must be the subtrahend. The difference is 17.” “I know that 3 + 17 = 20 so 20 – 3 = 17.”	Children can record multiple number sentences from one representation e.g. 20-3=17 20-17=3 3=20-17 17=20-3 Alongside teaching subtraction children should be taught the inverse operation within the same lesson.

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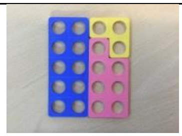
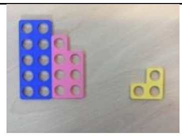
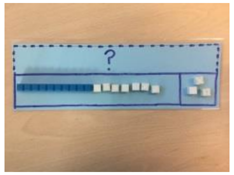
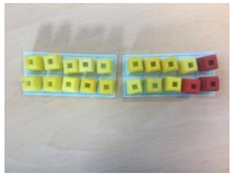
<td>  
 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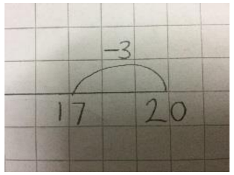
<td>

Children can also demonstrate this calculation on the number line. Emerging children will need to count back in ones rather than one jump of three.</td>
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<tr>
<td>Subtracting powers of 10 (10, 100, 1,000 etc.) or a multiple of 10

93 - ___ = 63

I have 93 pencils.63 of them are sharp. How many are blunt?

Using the vocabulary of 1 ten, 2 tens, 3 tens</td>
<td>Dienes apparatus allows children to see that ~~when~~ subtracting in powers of ten, certain columns remain unchanged. This builds up a conceptual understanding of subtracting in powers of 10.
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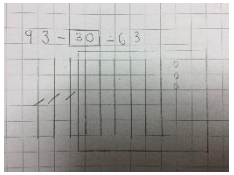
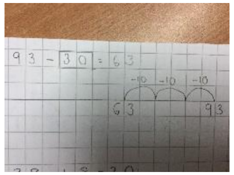
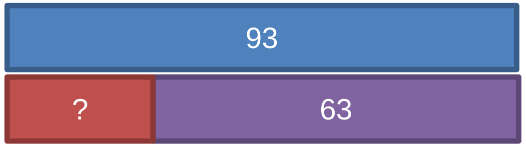
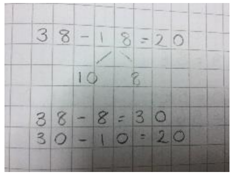
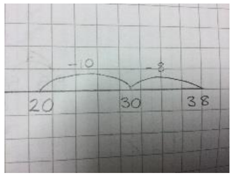
<td>“93 is the whole and 63 is apart. The other part is unknown. I need to add 30 to 63 to find the unknown part.”




When children are working with the part-part-whole diagram or the bar model, it allows children to be able to see the unknown and use counting on skills to work out the missing addend.</td>
<td>Children can record multiple number sentences from one representation e.g.

63=93-30
30= 93-63
93-63=30
93-30=63

Alongside teaching subtraction children should be taught the inverse operation within the same lesson. I have 63 pencils but I need 93. How many more do I need? I have 93 pencils in total. 63 are blue, how many are green?
63+__=93
93= 63 + __</td>
</tr>
</table>

etc., alongside 10, 20, 30 is important as pupils need to understand that it is a ten and not a one that is being subtracted. *It also emphasises the link to known number facts e.g. 9-3=6 so 9 tens – 3 tens = 6 tens.*			
## Subtracting to make a multiple of 10 38 – 18 = __ There are 38 cars altogether. 18 are blue. How many are red? *Children will be using their understanding of subtracting multiples of ten as well as their number bonds to subtract to make a multiple of 10 e.g. in this example children should take away 8 ones to make 0 ones and subtract 1 ten to make 2 tens. Children need to subtract the ones first as this links to formal written methods.*	Dienes apparatus and Numicon allows children to see that there will not be any ones left in the ones column because all 8 are being subtracted. They can then understand that this makes a multiple of ten.   Children could also use place value counters and bead strings to represent this calculation.	Partitioning allows children to see that the subtrahend can be broken down into 10 and 8 to easily subtract. Children should subtract 8 first and then 10 as this helps them move onto the column method when appropriate.  Children can also use the number line to subtract 8 ones to make a multiple of 10, and then the remaining 10 to find the difference. 	Children can record multiple number sentences from one representation e.g. 38-18=20 38-20=18 20=38-18 18=38-20 Alongside the concrete and the pictorial representations, children could explore column subtraction using a formal written algorithm. They should do it alongside the use of resources. $\begin{array}{r} 38 \\ \underline{-18} \\ \underline{20} \end{array}$

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Bridging through 10

14 – 6 = 8

Lily the giraffe has 14 spots. Elisha the giraffe has 6 less spots. How many spots does Elisha have?

In KS1 this is often referred to as the 'magic 10'. 'Magic 10' encourages the children to make 10 when bridging. In this case, children should partition the 5 into 4 and 1 to subtract the 4 to make 10 and then the 1 to make 9.

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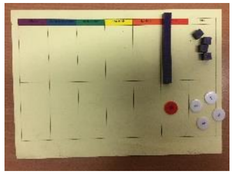
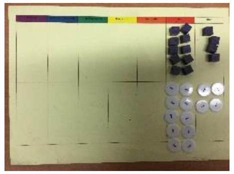
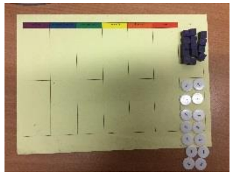
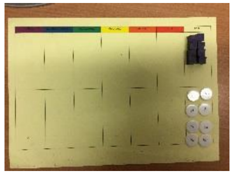
14 – 6 =?

Dienes and place value counters can be used to bridge through ten. Children need a secure understanding of renaming to be able to apply this concept. Children need to know that when you subtract 6 from 4, there are not enough ones so they need to rename one ten.

In addition to this, children could make 14 on the tens frame. Take away the four first to make 10 and then takeaway two more so you have taken away 6. You are left with the answer of 8.

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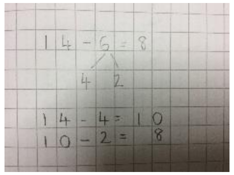
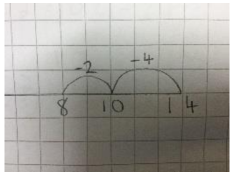
14 – 6 =

Partitioning allows children to see that the subtrahend can be broken down into 4 and 2 to easily subtract. Children should subtract 4 first and then 2 as this helps them to bridge through 10.



Children can also use the number line to subtract 4 ones to make a multiple of 10, and then the remaining 2 to find the difference.



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14 – 6 =

How many do we subtract to make 10?

14 – 4 = 10

How many do we have left to take off?

10 – 2 = 8

Children can then explain that altogether they have subtracted 6 because 4 and 2 equals 6.

Children do not need to use a formal written method for this strategy.

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Compensating to subtract.

95 – 39 =

95 bees live in a hive. 39 flew away. How many were left in the hive?
Children should still using compensation strategies with greater numbers e.g.
384-90 =
384-99=
21,587 – 9,990 =

21,587 people attended a football match. 9,990 of the fans were supporting the blue team. How many people supported the red team?

Children should use a near multiple of 10, 100, 1,000 or 10,000 and then readjust by compensating.
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These pictures demonstrate the use of dienes and place value counters to subtract. First the minuend is created. Then 40 is subtracted. Finally one is added back on to compensate.
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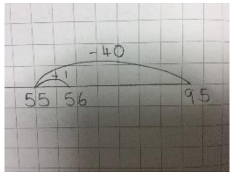
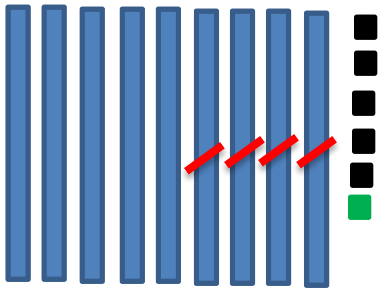
<td>
All pictorial representations show the same as the concrete representation. Forty has been subtracted from 95 and then one has been added on.

“I have subtracted 40 from 95 because 40 is 1 more than 39. I have then added 1 onto 55 to compensate for the extra one that I subtracted. The difference is 56.”

Children could show this on a number line or by drawing dienes.
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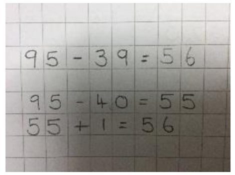
<td>
In children’s books, they could record the number equations as follows:


Children can record multiple number sentences from one representation e.g.

95-39=56
95-56=39
56=95-39
39=95-36
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<td>Find the difference by counting on.

79-63=

There are 79 beads in total. 63 are sparkly. How many are not sparkly?

Children should compare objects to find the difference e.g. using place value counters and counting on to find the difference.</td>
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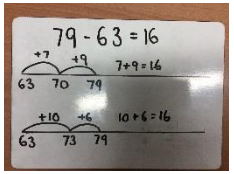
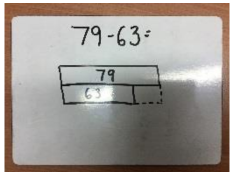
The bead string allows children to make 79 and slide 63 to one side. They are left with the beads that show the difference.</td>
<td>Children can draw a number line and count on to find the difference.



The bar model does not allow children to calculate the difference but it allows them to clearly see the difference. This allows them to see that they need to count on from 63 to 79 to find the difference. Children can do this by using the bridging through 10 strategy that they are already familiar with.


</td>
<td>Once children are ready, they can record a number sentence to match their picture e.g. 79-63=16

A written method can be used to calculate 79 – 63 = however the children should be able to fluently calculate this.

$$\begin{array}{r} 79 \\ -\ 63 \\ \hline 16 \\ \hline \end{array}$$</td>
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<td>Subtraction using partitioning with no renaming.

599 – 232 =

There were 599 children from two schools at a sports competition. 232 children from Sandown attended. How many children were from the other</td>
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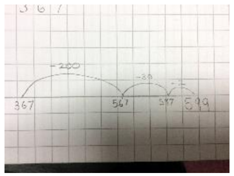
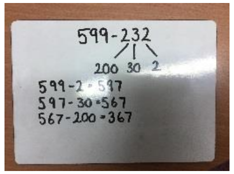
Many different resources could be used to exemplify this calculation. The abacus has been used to show 599 before 232 has been subtracted. The children should subtract from the ones, tens and then hundreds to match how they would use the formal written method.</td>
<td>Children could draw the dienes or place value counters alongside the written calculation to help to show working.

They could also use a number line to subtract or partition the subtrahend into ones, tens and hundreds to subtract.

 
</td>
<td>Children should only move onto a formal written method once they are secure on the concrete and pictorial. Children should use the side-by-side method to aid their understanding of the written method.

<u>Partitioned Column Method</u>

$$\begin{array}{r} 500 + 90 + 9 \\ -\ 200 + 30 + 2 \\ \hline 300 + 60 + 7 \end{array}$$

<u>Compact column method</u></td>
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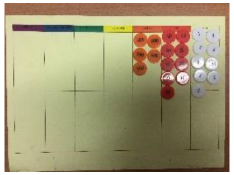
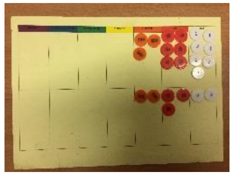
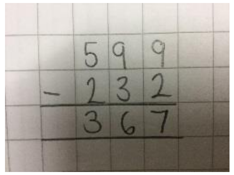
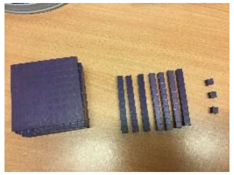
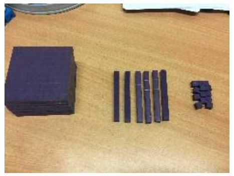
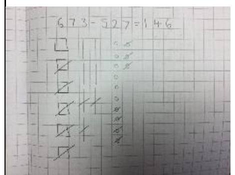
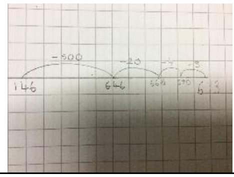
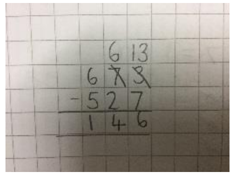
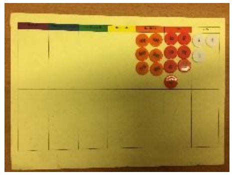
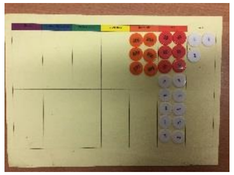
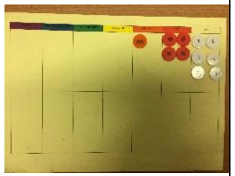
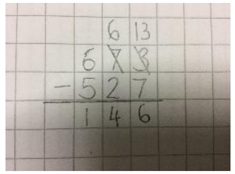
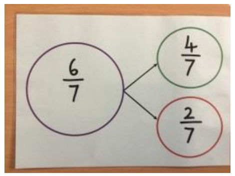
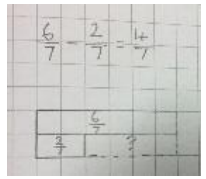
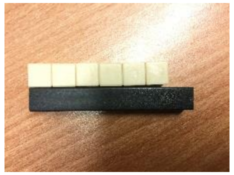
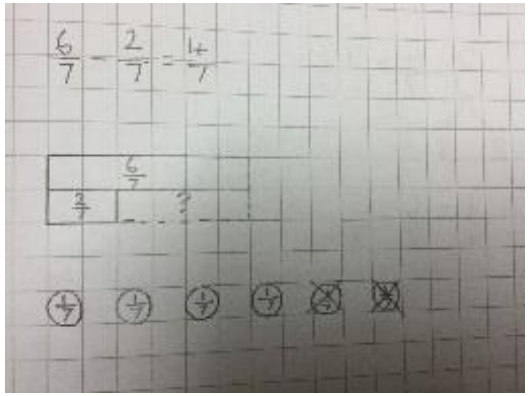
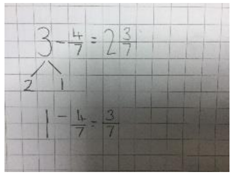
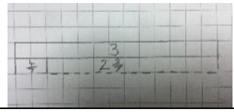
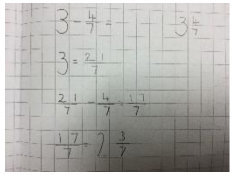
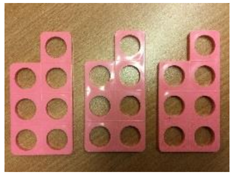
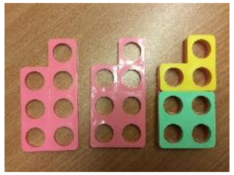
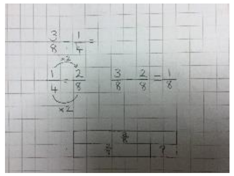
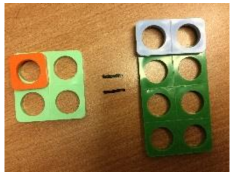
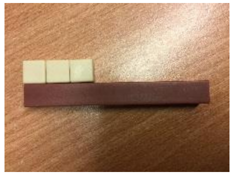
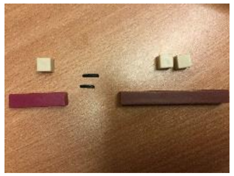
school? *In theory this is a mental strategy, however children need to be taught how to subtract without renaming by using the C-P-A approach. Once children are confident with this strategy, it should become a mental calculation.*	  Additionally, place value counters can be used to show the same calculation following the method used for the abacus.		This will lead to a clear written column subtraction. 
Subtraction with renaming. 673 – 527 = The total number of team points is 673. Yellow team gained 527 points. How many points did red team gain? *In this example children are renaming from tens to one. Children need to use concrete manipulatives alongside pictorial*	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.     Make the larger number with the place value counters	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup. 	Start by partitioning the numbers before moving on clearly show the subtraction strategy.  to Partitioned Column Method Compact Column Method Moving forward the children use a more compact method.

representations. *It is important for children to use the correct mathematical language when calculating using this strategy e.g. 3 ones subtract 7 ones is not possible so I need to rename 1 ten to make 13 ones. 13 ones subtract 7 ones equals 6 ones.*	   Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. Now I can subtract my ones. Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens. Now I can take away eight tens and complete my subtraction These images show the side by side approach to teaching the column method for subtraction. This links the enactive-iconic-symbolic modes of representation. Cross out the numbers when exchanging and show where we write our new amount.		This will lead to an understanding of subtracting any number including decimals. 
Subtracting fractions with the same denominator within one whole	For children to be able to subtract fractions within one whole, they need to show the minuend and then subtract the subtrahend to find the difference.	These pictorial representations replicate the concrete representation of subtracting fractions with the same denominator.	 

$\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$ $\frac{6}{7}$ of a cake was on a plate. Someone ate $\frac{2}{7}$. How much was left? *Pupils should be taught to practise subtracting fractions with the same denominator to become fluent through a variety of increasingly complex problem within one whole. Children should be using fluent recall of subtraction facts to calculate the total.*	The Numicon and Cuisenaire show that the denominator is sevenths and the numerator is 6. The Cusinennair is a clearer representation because 2 sevenths are easily subtracted. With the Numicon, you have to place a two plate over the 6 plate to demonstrate that the difference is 4 sevenths. *Children should be applying their number bonds (to 6 in this example) when subtracting fractions with the same denominator.*    	The bar model (which in this case is drawn using six squares to represent the 6 sevenths) shows that 6 sevenths is the whole and 2 sevenths are being subtracted from the whole. The dashed part of the bar represents the difference.  	These representations are abstract because nothing about the numbers within the bar model or part-part-whole model represent the value of the minuend or subtrahend. Children will record the number sentence alongside their pictorial representation in their books. $\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$ or $\frac{6}{7} - \frac{4}{7} + \frac{2}{7}$
Subtracting fractions from whole numbers $3 - \frac{4}{7} =$ There are 3 chocolate bars. 4 children eat $\frac{1}{7}$ of a	For children to be able to subtract fractions from one whole, they need to show the minuend and then subtract the subtrahend to find the difference. Children need to choose either the 7 plate or the 7 Cuisenaire rod because they are subtracting sevenths. This 	 In this example, the 3 has been partitioned using a part-whole diagram into 2 and 1. Children have then applied their understanding of subtracting a fraction from 1 whole to find the difference. 	 In this abstract example, 3 is converted into $\frac{21}{7}$ to allow $\frac{4}{7}$ to be subtracted fluently. The improper fraction $\frac{17}{7}$ has then been converted into a mixed number to

bar. How much is left? *Children should recognise that $\frac{4}{7}$ is less than one whole. Therefore, they know that one whole is equivalent to $\frac{7}{7}$ so $\frac{7}{7} - \frac{4}{7} = \frac{3}{7}$ meaning that $3-\frac{4}{7}=2\frac{3}{7}$. Alternatively, children could convert 3 into an improper fraction and subtract $\frac{4}{7}$ to find $\frac{17}{7}$ and then convert it back into a mixed number.*	relies on their understanding that 7 sevenths is one whole so 21 sevenths is 3 wholes.   Cuisenaire - Children need to exchange one whole bar for 7 sevenths (seven ones). They can then easily subtract 4 sevenths to show the difference is $2\frac{3}{7}$. Numicon- Children can use the 4 plate to cover up part of 1 whole to show that this has been subtracted. They can then see that the difference is $2\frac{3}{7}$.   *Children should be applying their number bonds (to 7 in this example) when subtracting fractions from whole number.*	This bar model shows that 3 is the whole and $\frac{4}{7}$ is the subtrahend. It is clear to see that the difference is the dashed part of the bar model. This bar model does not solve the calculation but pictorially demonstrates what maths the children need to calculate.	show the difference.
Subtracting fractions using the same denominator or multiple of the	When children move onto subtracting fractions with different denominators, the concrete resources become more abstract because children need to convert the fractions into common denominators before	$- \frac{1}{4} (\frac{2}{8})$ $\frac{1}{8}$ $\frac{3}{8}$	

same denominator $$\frac{3}{8} - \frac{1}{4} =$$ $\frac{3}{8}$ of the class wear glasses. $\frac{1}{4}$ of these are boys. How many girls wear glasses? *Children need to learn that the most efficient way of subtracting fractions, would be to make the denominators equivalent. They would need to convert $\frac{1}{4}$ into $\frac{2}{8}$ and then fluently subtract.*	they can calculate e.g. in this case eighths is the common denominator $\frac{1}{4} = \frac{2}{8}$.   The Numicon demonstrates that $\frac{1}{4} = \frac{2}{8}$. $\frac{3}{8}$ has been created using Numicon and then 2 eighths has been subtracted. It is now clear that $\frac{1}{8}$ is the difference.    The Cuisennaire demonstrates that $\frac{1}{4} = \frac{2}{8}$. $\frac{3}{8}$ has been created using	A good pictorial representation to use is a number line. Children need to know that $\frac{1}{4} = \frac{2}{8}$ before they can calucalte using this representation.	In this abstract example, $\frac{1}{4}$ is converted into $\frac{2}{8}$ to allow children to subtract from $\frac{3}{8}$. Children should display the conversion arrows and annotate them to show the multiplicative relationship. This builds on their fluency.

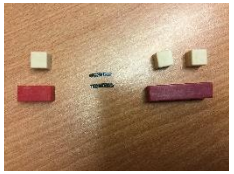
<table>
<tr>
<td></td>
<td>Cuisenaire and then 2 eighths has been subtracted. It is now clear that $\frac{1}{8}$ is the difference.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtracting fractions with different denominators or mixed numbers.

$$3\frac{3}{4} - 2\frac{1}{2} =$$

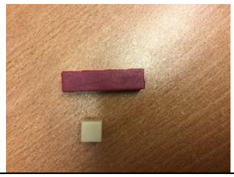
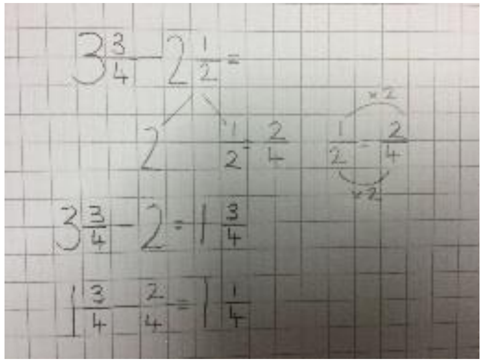
It took 3 $\frac{3}{4}$ hours to make a cake. I spent $2\frac{1}{2}$ hours baking the cake. How long did it take me to ice the cake?

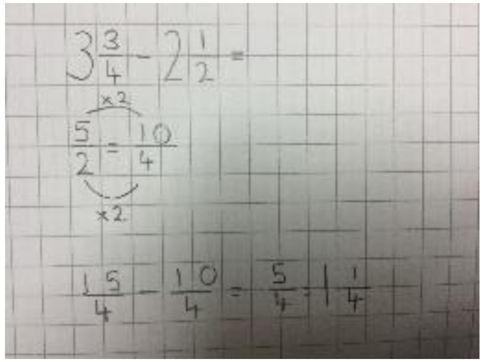
Children need to have a secure understanding of equivalence as they will need to identify equivalent fractions with the same denominator.</td>
<td>When calcuations move into higher-order thinking, the concrete becomes the abstract. It is easier for children to represent this calcuation by using pictorial or abstract representations.

The Cuisennaire demonstrates that $\frac{1}{2} = \frac{2}{4}$. $3\frac{3}{4}$ has been created using



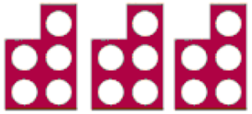
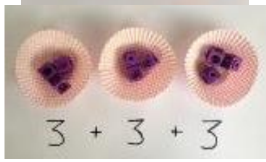
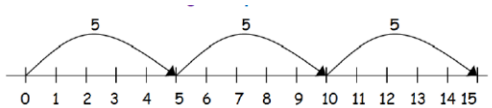
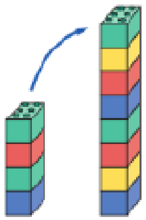
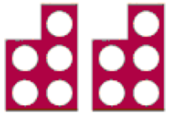
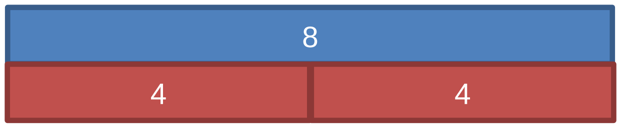

Cuisenaire and then $2\frac{2}{4}$ has been subtracted. It is now clear that $1\frac{1}{4}$ is the difference.

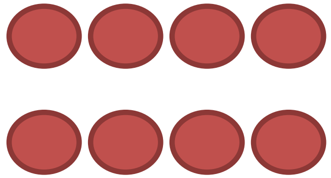
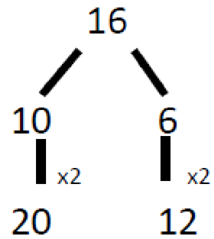
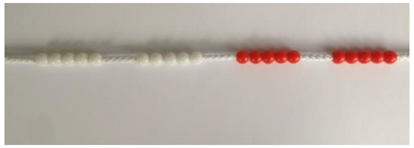
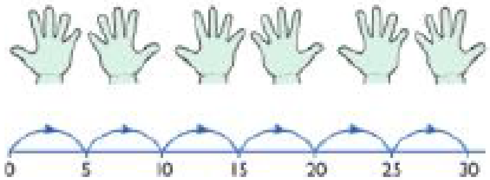
</td>
<td>

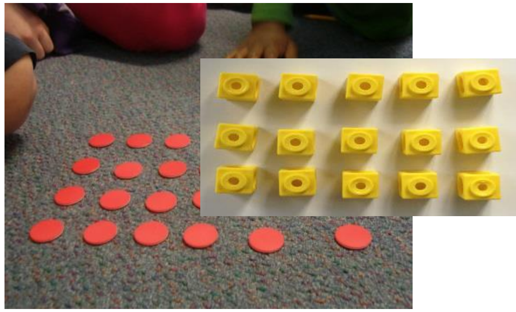
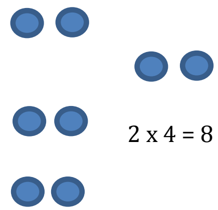
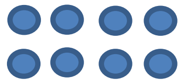
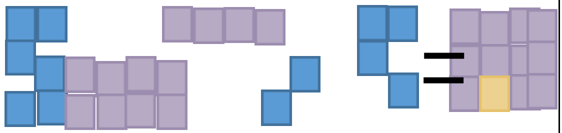
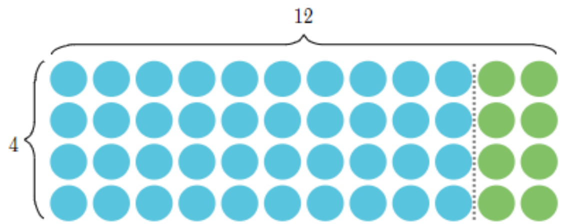
In this example, the $2\frac{1}{2}$ has been partitioned using a part-whole diagram into $2\frac{2}{4}$ because the child has worked out that this is equivalent to $2\frac{1}{2}$. First, the child has subtracted the two wholes to find the difference of $1\frac{3}{4}$ and then subtracted the $\frac{2}{4}$. They have then subtracted the whole subtrahend. This shows that the difference is $1\frac{1}{4}$.</td>
<td>

In this abstract example, the mixed numbers have been converted into improper fractions. $\frac{5}{2}$ has been converted into $\frac{10}{4}$ to give the fractions the same denominator. $\frac{10}{4}$ can then be subtracted from $\frac{15}{4}$ to find the difference of $\frac{5}{4}$. $\frac{5}{4}$ is then converted back into a mixed number to show that the difference is $1\frac{1}{4}$.</td>
</tr>
</table>

Multiplication

Objective and Strategies	**Concrete** Children should be taught to use a range of resources to represent one calculation. Children should verbalise their use of resources using the appropriate mathematical language.	**Pictorial** Children should be taught to draw pictorial representations independently. Children can use different colours or symbols to distinguish between the different parts in the number sentence.	**Abstract** The abstract should be recorded alongside the concrete and pictorial.
Repeated addition (& arrays)	Use different objects to add in equal groups.    	Use a number line or pictures to continue to support repeated addition.   5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Doubling *(show as repeated addition or multiplication – see grid in the appendices)*	Use practical activities to show how to double a number.  Double 4 equals 8 4 + 4 = 8 4 x 2 = 8  Double 5 equals 10 5 + 5 = 10 5 x 2 = 10	Draw pictures to show how to double a number. Double 4 is 8  Bar Model  	Children can record doubling in three different ways. 1. Children can write 'double 4 equals 8'. 2. Children can show it as repeated addition '4 + 4 = 8' 3. Children can show it as multiplication '4 x 2 = 8'

		Arrays 	When doubling two digit numbers, partition a number and then double each part before recombining it back together. 
Counting in multiples	Count in multiples supported by concrete objects in equal groups.  	Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 5, 10, 15, 20, 25 , 30

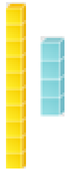
Commutative multiplication To understand that the order of the multiplication does not affect the answer. (in addition to this children learn to multiply using associative law – multiplying 3 digits) The children learn that they can multiply in any order **Associative law** **a x b x c = (a x b) x c** = a x (b x c))	Create arrays using counters/ cubes to show multiplication sentences. 	Draw arrays in different rotations to find commutative multiplication sentences.  2 x 4 = 8 4 x 2 = 8 Link arrays to area of rectangles.  4 x 2 = 8 2 x 4 = 8	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Multiplying of number using distributive law *Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations. As well as partitioning into tens and ones (a familiar strategy), they begin*	You can use dienes, counters etc. to illustrate this using arrays. 3 × (2 + 4) = 3×2 + 3×4  3 x (2+4) = 3 x 2 + 3 x 4	We can use the distributive law to help with multiplication calculations, for example 4 x 12= Change 4 x 12 into 4 x (10 +2) The 4 gets distributed to the 10 and 2 and changes to (4 x 10) + (4 x 2)  4 x 10 + 4 x 2	Multiplication is distributive over addition and subtraction, e.g. (50 + 6) × 4 = (50 × 4) + (6 × 4) and (30 – 2) × 4 = (30 × 4) – (2 × 4). 6 × 204 = 6 × 204 = 6×200 + 6×4 = 1,200 + 24 = 1,224

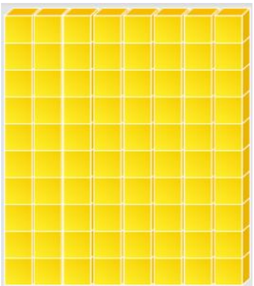
to explore compensating strategies and factorisation to find the most efficient solution to a calculation. **Distributive law** **a x (b + c) = a x b + a x**		Now add the expressions to find the total (4 x 10) + (4 x 2) = 40 + 8 = 48	

Multiplying by 10, 100 and 1000

Understanding of place value allows children to make use of known facts.
Use times tables that they are familiar with.

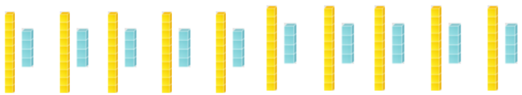
8 x 10=
Children use base 10 to count in 10s





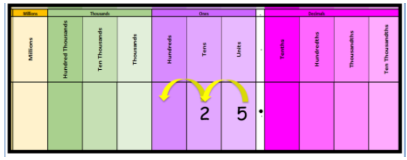
1 block of 10 -

14 x 10 =



14 + 14 + 14 + 14 + 14 14 + 14 + 14 + 14 + 14

25 x 10=
Multiplying by 10- move 1 plave to the left



<u>25 X 10 = 250</u>

25 X 100=
Multiplying by 100- move 2 places to the left

25 X 100 = 2500

25 x 1000 =
Multiplying by 1000- move 3 places to the left

When we multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds and so on.

25 x 10 =250

When multiplying whole numbers, a zero becomes a place holder so that each digit has a value that is ten times greater.

8 x 10 =80

14 x 10 =140

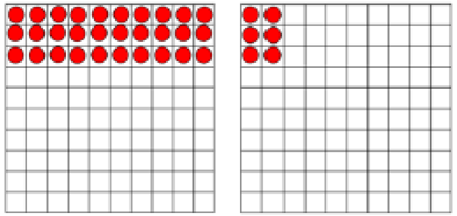
25 x 10 = 250

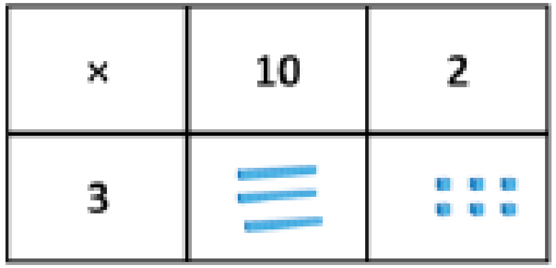
25 x 100 = 2500

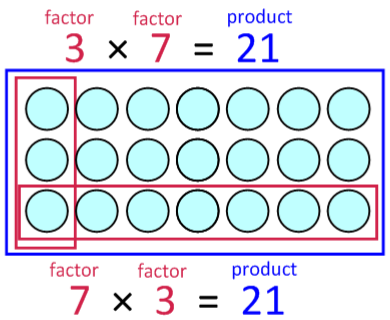
25 x 1000 = 25000

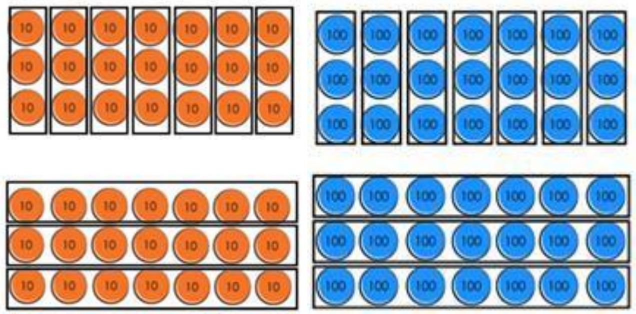
<table>
<tr>
<td>Multiplying 2 digit number by 1 digit number using partitioning

Pupils to use the most efficient strategy</td>
<td>3 x 12
12 = 10 + 2

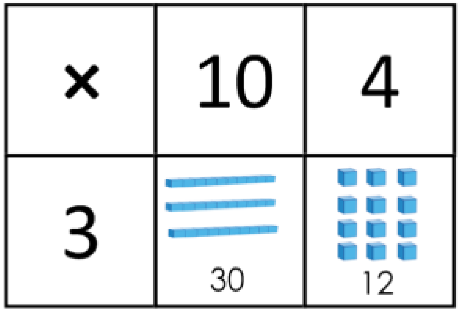
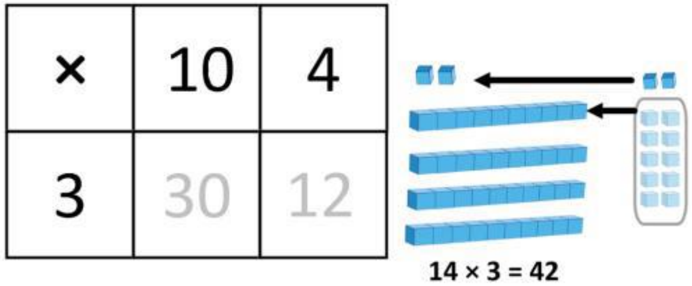
3 X 10   3 X 2


Now add the total number of tens and ones</td>
<td><table><tr><td>×</td><td>10</td><td>2</td></tr><tr><td>3</td><td>30</td><td>6</td></tr></table>
</td>
<td>3 x 12
10 and 2 make 12
3 x 2 = 6
3 x 10 = 30
30 + 6 = 36</td>
</tr>
<tr>
<td>Multiplying 2 digit number by 1 digit number without partitioning

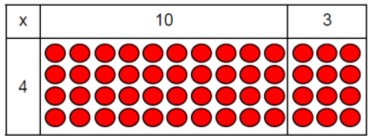
Pupils to use the most efficient strategy e.g. known facts</td>
<td>

7X3 = 21 then multiply by 10 for 70x3</td>
<td></td>
<td>30 x 7 = 210   300 x 7 = 2100
70 x 3 = 210   700 x 3 = 2100
7 x 30 = 210   7 x 300 = 2100
3 x 70 = 210   3 x 700 = 2100</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>or 7x30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplying using regrouping

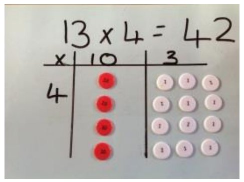
Secure knowledge of place value allows the children to regroup in any column.</td>
<td>
</td>
<td>
</td>
<td>14 x 3 = 42

10 x3 = 30
4 x 3 = 12</td>
</tr>
<tr>
<td>Grid Method

Using partitioning to solve multiplication problems</td>
<td>**<u>2 digit by one digit</u>**
Show the link with arrays to first introduce the grid method.
13 x 4=


4 rows of 10
4 rows of 3</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.

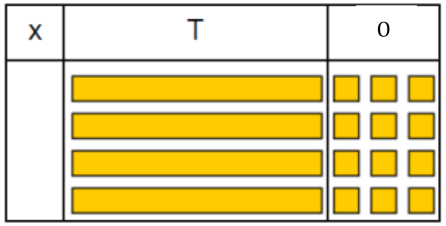
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

<u>2 digit by one digit</u>

</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

<u>2 digit by one digit</u>
<table><tr><td>X</td><td>10</td><td>3</td></tr><tr><td>4</td><td>40</td><td>12</td></tr></table>
40+12 = 52</td>
</tr>
</table>

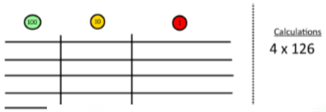
Move on to using Base 10 to move towards a more compact method.

4 rows of 13

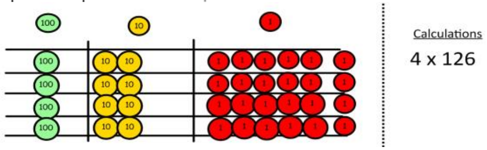


Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.

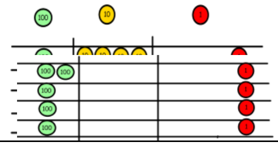
3 digit by one digit



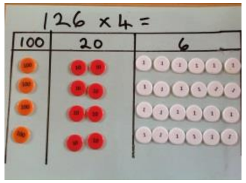
Fill each row with 126.



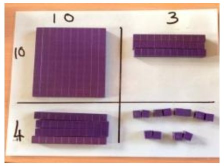
Add up eac h column, starting with the ones making any exchanges needed.



3 digit by one digit



2 digit by two digit



3 digit by one digit

X	100	20	6
4	400	80	24

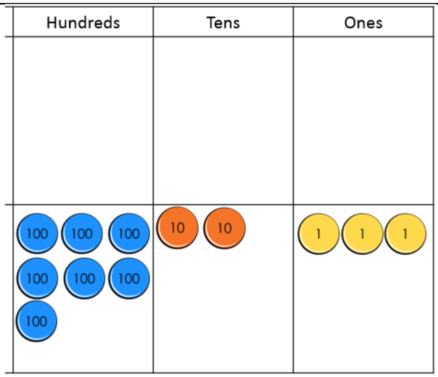
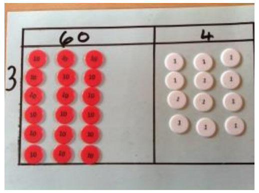
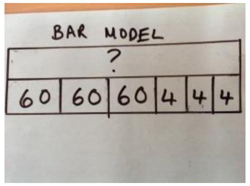
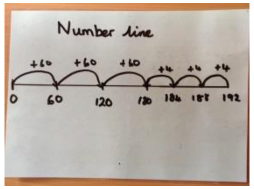
400+80+24 = 504

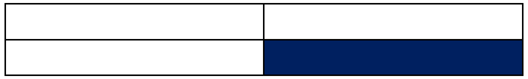
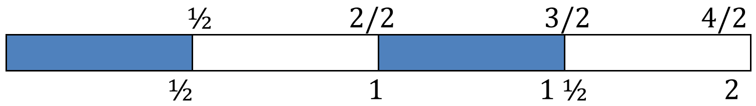
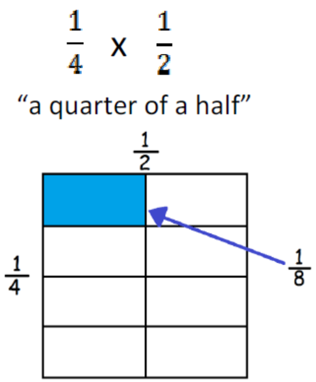
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

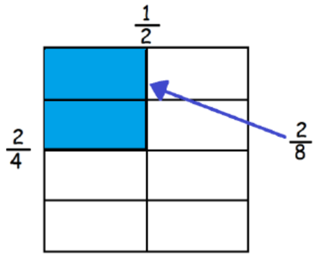
2 digit by 2 digit

X	10	3
10	100	30
4	40	12

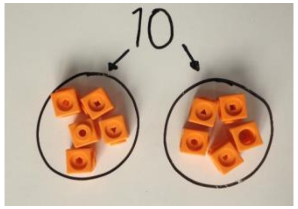
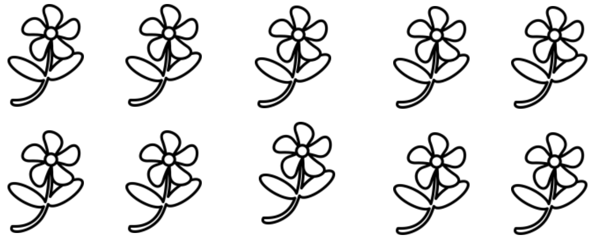
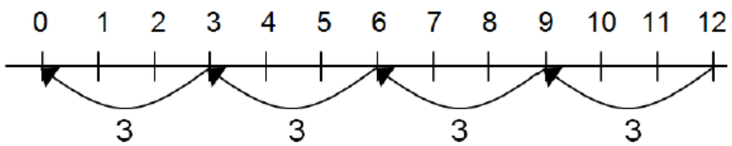
100 + 30 = 130
40 + 12 = 52
130 + 52 = 182

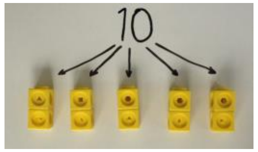
	Then you have your answer.		
Short multiplication		To calculate 241 x 3, represent the number 241. Multiply each part by 3, regrouping as needed. 	2 4 1 x 3 7 2 3 1
Column multiplication (Formal written method of short multiplication)	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 64 <u>X3</u> 12 (3X4)

			180 (3X60) 192 This moves to the more compact method.
Multiplying fractions by whole numbers Product of two fractions – multiplying two fractions is the same as finding the fractional part of another fraction.	 ½ X 3 = 1 ½	 ½ X 3 = 1 ½	$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$
Multiply simple pairs of proper fractions, writing the answer in its simplest form Product of two fractions – multiplying two fractions is the same as finding the fractional part of another fraction. – in simplest form		Picture grid 	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

		 $\frac{2}{8} = \frac{1}{4}$	
Multiply decimals in the context of money using concrete resources and repeated addition. £2.45 x 3 = £7. 35	Children to use money	 £6 + £1.20 + 15p	£2.45 x 3 = £7.35

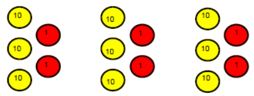
Division

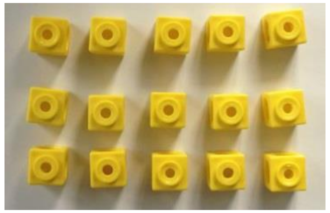
Objective and Strategies	**Concrete** Children should be taught to use a range of resources to represent one calculation. Children should verbalise their use of resources using the appropriate mathematical language.	**Pictorial** Children should be taught to draw pictorial representations independently. Children can use different colours or symbols to distinguish between the different parts in the number sentence.	**Abstract** The abstract should be recorded alongside the concrete and pictorial.
Sharing objects into groups Keep practical - division symbol is not formally taught.	I have 10 cubes, can you share them equally in 2 groups?   	Children use pictures or shapes to share quantities.  $10 \div 2 = 5$	Share 10 buns between two people. $10 \div 2 = 5$
Division grouping (arrays)	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?



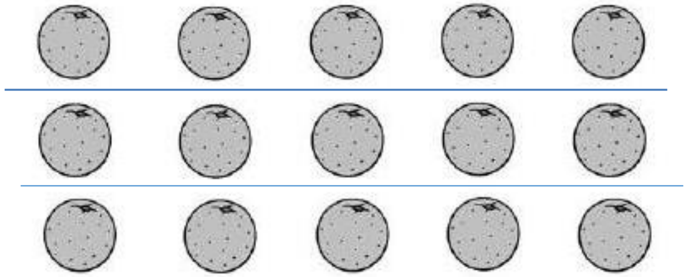
Link division to multiplication by creating an array and thinking about the number sentences that can be created.

96 ÷ 3 = 32

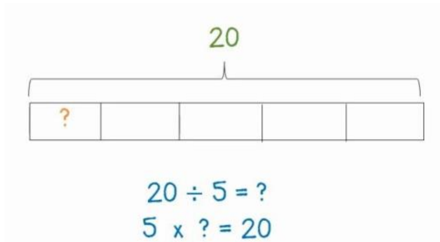




15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15

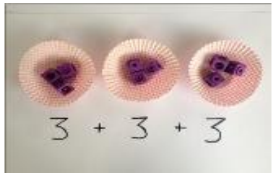
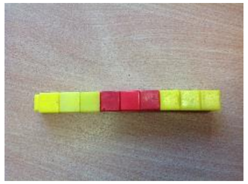
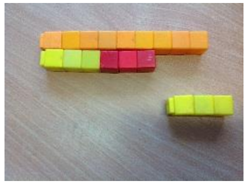
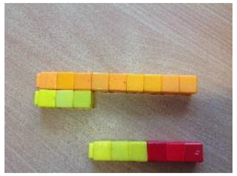
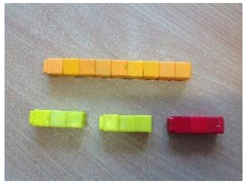
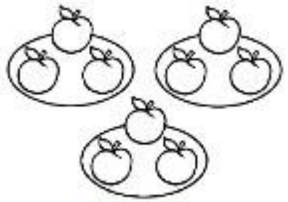


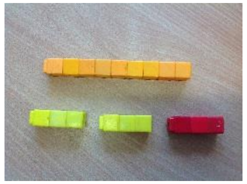
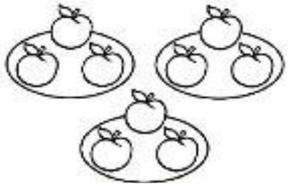
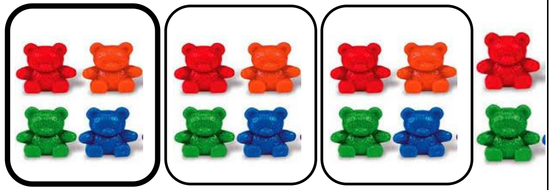
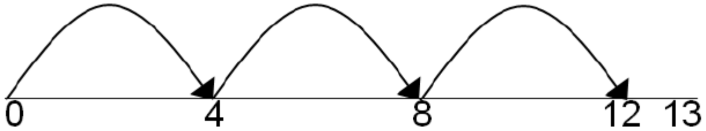
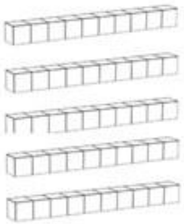
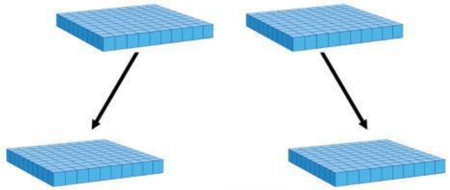
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

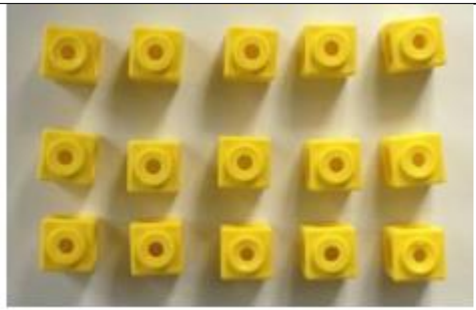
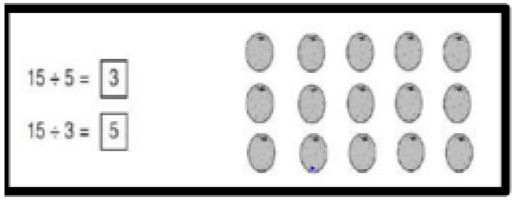
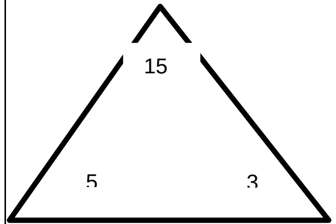
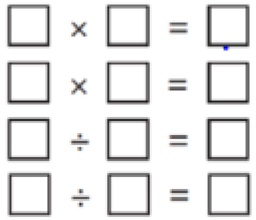
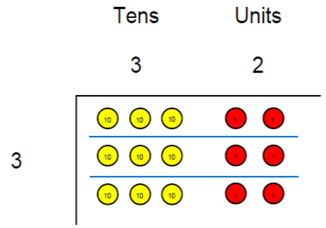
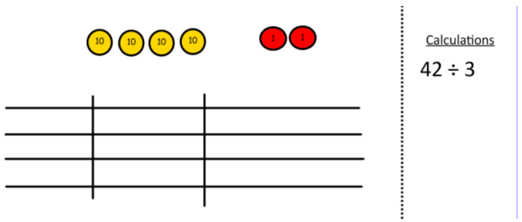
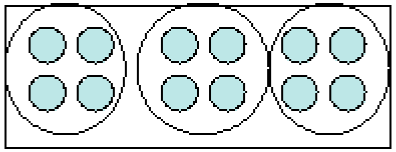


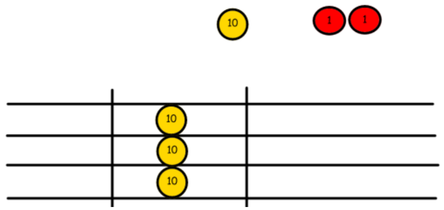
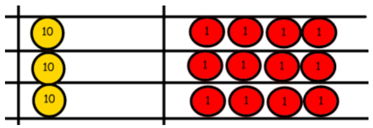
Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

Division with repeated subtraction/ Addition	     9 subtract 3 subtract 3 subtract 3		3 + 3 + 3 = 9 9 - 3 -3 -3 = 0

Dividing by using multiplication facts for division		 3 X 3 = 9 9 ÷ 3 = 3	3 X 3 = 9 9 ÷ 3 = 3
Division with a remainder Use part, part whole method to demonstrate	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder
Division within 10, 100, 1000 *When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller*	50 divided by 5 groups of 10 	 200 ÷ 100 = $\boxed{2}$	200 ÷100 =2

Division using known facts understanding the inverse relationship between multiplication and division		 	 
Short division Dividing 4 digit by 1 digit numbers (use concepts of sharing and grouping)	 Use place value counters to divide using the bus stop method alongside  Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 | 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 | 4 3 2

	 We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 | 5 1 1 . 0

<table>
<tr>
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Long division

Dividing 3 and 4 digit by 2 digits

Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.

</td>
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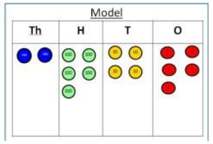
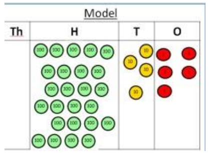
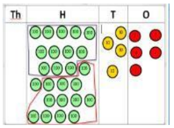


0 2 1 2
12 | 2544
24
14
12
24
24
0

2544 ÷ 12 How many groups of 12 thousands do we have? None



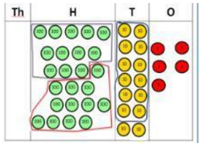
Exchange 2 thousand for 20 hundreds.



0 2
12 | 2544
24
1

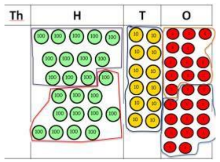
How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



0 2 1
12 | 2544
24
14
12
2

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 2544?

</td>
<td>

Children to represent the counters, pictorially and record the subtractions beneath.

</td>
<td>

0
12 | 2544

Step one- exchange 2 thousand for 0 hundreds so we now have 25 hundreds.

0 2
12 | 2544
24
1

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

0 2 1
12 | 2544
24
14
12
2

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.

0 2 1 2
12 | 2544
24
14
12
24
24
0

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

</td>
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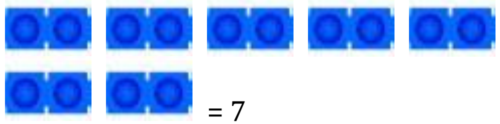
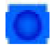
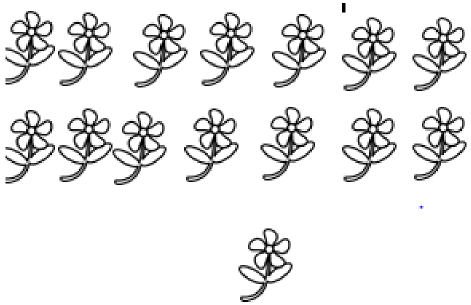
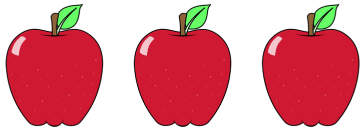
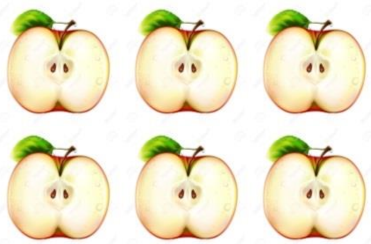
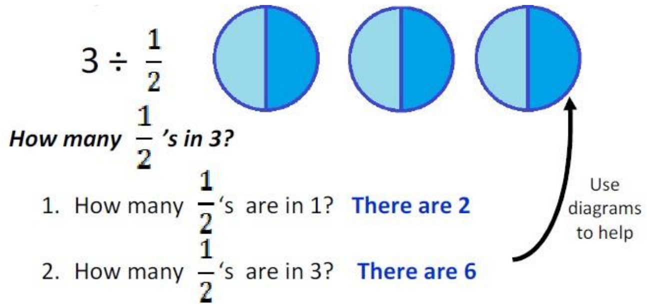
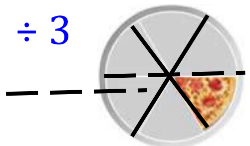
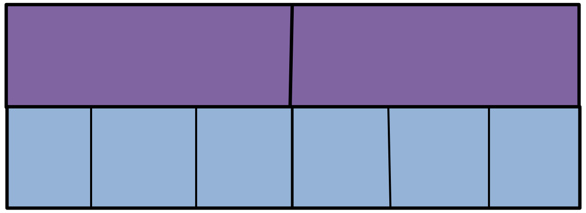
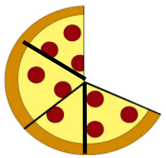
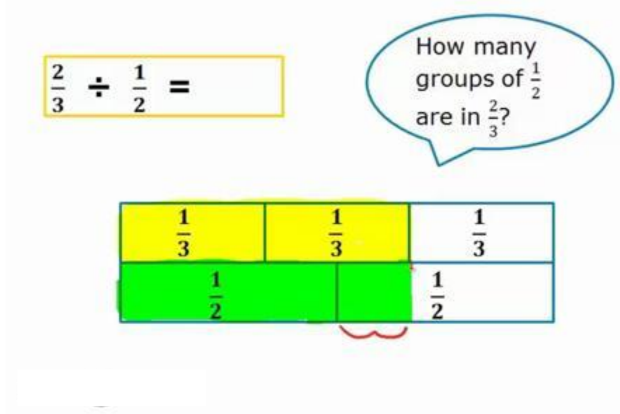
Remainders as fractions or decimals	15÷2 = Divide objects between groups and see how many are left out of 2   = 7 1 left from a group of 2 	15 into groups of 2 = 7 with one out of two remaining 	Complete written divisions and show the remainder using as the numerator and the divisor as the denominator $15 \div 2 = 7\frac{1}{2}$
Dividing whole numbers by proper fractions.	3 ÷ ½ = 6  CUT IN HALF 		3÷ ½ = Flip ½ to its reciprocal $\frac{2}{1}$ $\frac{3 \times 2}{1 \times 1} = \frac{6}{1} = 6$

Divide proper fractions by whole numbers e.g. 1/2 divided by 3= 1/6	$\frac{1}{2} \div \frac{3}{1} =$  ½ ÷ 3 	$\frac{1}{2} \div \frac{3}{1} =$ 	$\frac{1}{2} \div \frac{3}{1} =$ Flip $\frac{3}{1}$ to its reciprocal $\frac{1}{3}$ $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
Dividing proper fractions by proper fractions	$\frac{2}{3} \div \frac{1}{2} =$ How many ½ slices fit into a 2/3 slice?  in 		$\frac{2}{3} \div ½ =$ Flip ½ to its reciprocal $\frac{2}{1}$ $\frac{2 \times 2}{3 \times 1} = \frac{4}{3}$ $\frac{4}{3} = 1$ and $\frac{1}{3}$

Addition

addend + addend = sum

sum = addend + addend

sum	
addend	addend

Subtraction

minuend – subtrahend = difference

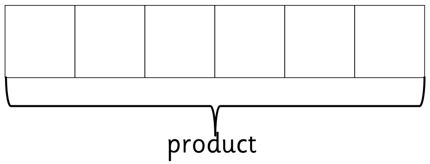
difference = minuend - subtrahend

minuend	
difference	subtrahend

Multiplication

multiplicand x multiplier = product

factor x factor = product



Division

dividend ÷ divisor = quotient

quotient = dividend ÷ divisor